

## ONS Problem Set 8

Wednesday, January 31, 2018

### Problem 1: Channel capacity

- a) Calculate the entropy  $H$  of a binary set  $\{0; 1\}$  for which the probabilities are given as  $p(0) = 1 - p(1)$ . Calculate the maximum of  $H$ ; what values do  $p(0)$  and  $p(1)$  take in that case?
- b) You are given the alphabet  $\{0; 10; 110; 111\}$  and the probabilities
  - i.  $\{0.25; 0.25; 0.25; 0.25\}$
  - ii.  $\{8/15; 4/15; 2/15; 1/15\}$
 Calculate the entropy of the source for these cases and compare it to the mean word length.
- c) In a practical system, nonlinear interference limits the capacity of optical networks after a certain launch power. Considering a WDM system, describe in words the three major nonlinear effects associated with the Kerr nonlinearity.

### Problem 2: Forward Error Correction (FEC) – Hamming Code

Bit errors in data signals can be corrected by adding a suitable overhead. Here, we use a (7,4)-Hamming code in which three overhead bits are added to every four input bits.

Assume that you have received the bit streams “0110011” and “1011010”. Check if the bit sequences have been received correctly, or otherwise correct them. Use the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

And the parity check matrix

$$H^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$